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The theoretical prediction of the cracking stress of glass fibre reinforced inorganic cement

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Glass fibre-reinforced inorganic cement is increasingly being seen as a promising new building material. For any structural use, its cracking stress will be significant because of its resemblance to the yield point of mild steel. In this paper, the use of fracture mechanics to formulate a prediction of the cracking stress is proposed. Experimental evidence is also presented in support of this theory.

1. Introduction

Glass fibre-reinforced inorganic cement is now recognized to be a promising new building material. In a previous article [1] the authors demonstrated that the strength of glass fibre-reinforced high-alumina cement did not deteriorate with time owing to a possible attack on the glass fibres by alkalis of the cement paste as had been suspected. The Building Research Station has recently developed an alkaline resistant glass [2] which can be used with Portland cement. Thus the durability of glass fibre-reinforced cement as a building material is ascertained.

In the previous article [1] the authors attempted to explain the behaviour of the composite under tension. With reference to the stress-strain diagram in Fig. 1, it was pointed out that the composite behaved linearly elastically in region OA, while in region ACD there was a certain amount of irreversible deformation. It was postulated that initially in region OA the fibres acted as crack arrestors, restraining the internal flaws from propagating, and at the same time sharing the load with the cement matrix. In region AC the initial microcracks started to propagate and there was a gradual transfer of load from the matrix to the fibres. This process ceased at point C, resulting in a reduction in the elastic modulus, and from then on the matrix served only to transfer load and maintain positional geometry. At the ultimate tensile stress the strain is of the order of 1 %.

From the above description of the behaviour

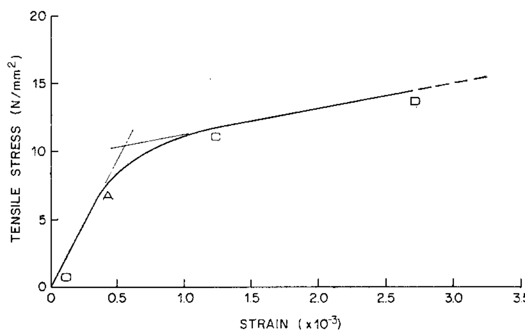

Figure 1 Stress-strain relationship of glass fibre-reinforced cement.

of the composite, it is obvious that the governing value of stress in the design of any structural component would be the cracking stress at point A of the stress-strain diagram, rather than the ultimate tensile stress, since beyond A irreversible deformation due to propagation of cracks occurs. Thus the region ACD with relative high ultimate tensile stress should be viewed as a safety reserve giving the material pseudo-plasticity and increased fracture toughness. The cracking stress at A resembles, to a certain extent, the yield point and the region ACD, the plastic and strain-hardening behaviour of mild steel.

In this paper, a theoretical prediction based on fracture mechanics of the cracking stress of fibre-reinforced inorganic cement is proposed. Experimental evidence to support this hypothesis is also presented.

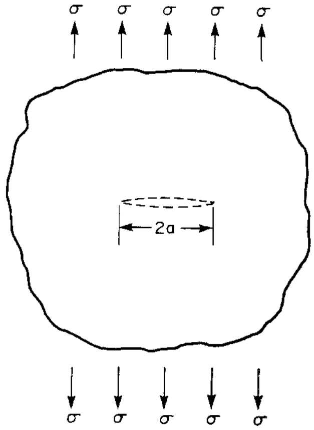

Figure 2 Crack in material under tension.

2. Theoretical basis

As has been stated, the role of the fibres is initially to restrain cracks, inherent in the matrix, from premature propagation. To understand the mechanism of crack arrest some knowledge of fracture mechanics is required. It is not the intention of this paper to review this subject since it can be found in a number of articles elsewhere [3, 4].

Consider an internal crack of radius a in a material subjected to tensile stress σ, Fig. 2, the crack extension force G, which is dependent on the loading system and geometry can be expressed as

$$G = \frac{4a\,\sigma^2}{\pi E} \qquad (1)$$

where E is the elastic modulus of the material. The critical value G_c is a material constant as the elastic modulus. When G approaches the critical value G_c of the material the crack becomes unstable and fracture occurs. Thus G_c is a measurement of the ability of a material to resist fracture.

This quantitative value of toughness may also be expressed by the intensity factor K [5]. The relation between K and G in a tensile stress field is

$$K^2 = \frac{G}{E} \qquad (2)$$

The advantage of using the stress intensity factor is the application of the principle of superposition if more than one stress field is applied to the material.

$$K_T = K_1 \pm K_2 \pm K_3 \ldots \pm K_i \qquad (3)$$

in which K_T is the total stress intensity factor and K_i, that associated with a particular loading system i.

Romaldi and Batson [6] showed that the tensile strength of concrete was greatly increased when reinforced with closely spaced small steel wires. They further investigated theoretically [4] the crack arresting effect of wire reinforcements on a disc shape crack of diameter equal to the spacings of the wire, Fig. 3. The stress intensity factor approach was used and solutions were obtained by numerical analysis assuming rigid

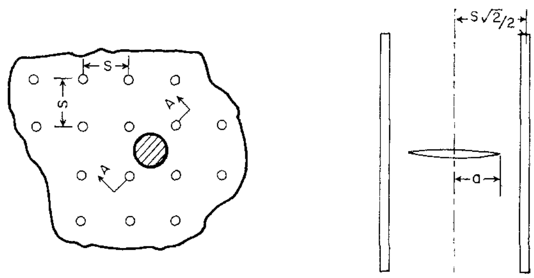

Figure 3 Reinforcement on a disc-shaped crack.

reinforcement. This assumption is justifiable as steel has a much higher elastic modulus than that of concrete. The crack-arresting phenomenon can be viewed as a negative pressure exerted by the internal forces of the wires on the crack surfaces thus tending to close the crack.

In the present case of glass fibre-reinforced inorganic cement, since the elastic moduli of both constituents are of the same order, the internal forces in the fibres will have very little effect on crack arrest. It is proposed here that, in this type of composite system, the presence of the fibres physically limits the possible size of the initial flaws of the brittle matrix. For example if s is the spacing of parallel fibres in the matrix, the maximum diameter of a disc-shaped crack perpendicular to the fibres will be $\sqrt{2}s$. Referring to Equation 1, the stress at which cracks start to propagate, σ_0, will be

$$\sigma_0 = \sqrt{\frac{\pi\,E\,G_c}{2\sqrt{2}s}} \qquad (4)$$

in which E and G_c are the material properties of the matrix. Thus Equation 4 can be used to predict the cracking stress of a brittle material reinforced with fibres of low elastic modulus provided E, G_c of the matrix are known.

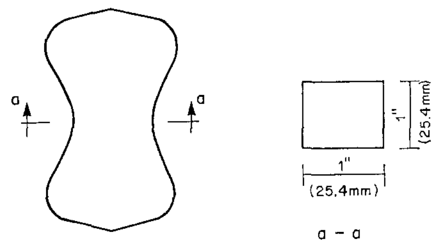

Figure 4 Briquette specimen.

3. Experimental work

To support the proposed hypothesis, an experimental programme was carried out to test the effect of the spacing of reinforcements on the cracking stress in tension. The difficulty in obtaining pure tension in tensile test of brittle materials such as concrete and inorganic cement is well recognized. For expediency the type of specimen chosen for this series of tests was the figure-eight briquette, Fig. 4, the standard test for tensile strength of cement-sand mortar. The glass fibres were kept in position before casting by threading through holes on two templates, which were drilled at pre-determined spacing. Specimens reinforced with spacings of 0.075, 0.1, 0.125, 0.15, 0.175, and 0.2 in. (1.91, 2.54, 3.18, 3.81, 4.44, and 5.08) mm were made. High alumina cement with a water-cement ratio of 0.4 was used and all specimens were tested at 9 days. To detect the cracking stress, electric resistance type of strain gauges, 10 mm long, were stuck on both top and bottom sides of the briquettes to trace the stress-strain relationship. Values of cracking stress obtained are shown in Fig. 5.

In order to use Equation 3 to determine the theoretical cracking stress, E and G_c of the matrix must first be found. It was decided to use the same type of specimens for this purpose. The modulus of elasticity was obtained from the stress-strain relationship of plain cement specimens and was found to be 3×10^6 lb/in.^{2} (20.64 kN/mm^2). The determination of G_c was more complicated. In the present work the method proposed for testing of high strength metals [7] was used. A fine saw cut of depth a, was first introduced at one side of the specimen, Fig. 6, before testing in tension. It was assumed as in previous tests that tensile stress distributes uniformly across the section at the narrowest part of the specimen. The calibration for uniform tension is represented by the following equation [7]

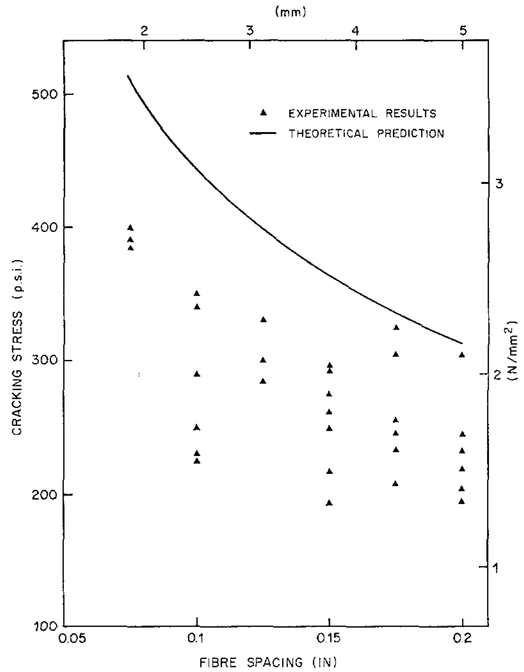

Figure 5 Cracking stress of fibre-reinforced briquette specimens.

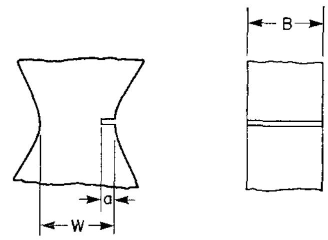

Figure 6 Fracture-toughness test specimen.

$$Y = K_c BW/Pa^{\frac{1}{2}} = 1.99 - 0.41\,(a/W) + 18.70\,(a/W)^2 - 38.48\,(a/W)^3 + 53.85\,(a/W)^4 \qquad (5)$$

where B is the depth, W, the width of the cross section of the specimen and P the load at rupture. The a/W ratio used was 0.2. Applying experimental data to Equations 5 and 2, the values of G_c, which have an average of 0.006 lb/in. (1.05 N/m), were obtained.

With the E and G_c values the theoretical cracking stress, Equation 4, is represented in Fig. 5 by the full line. This agrees fairly well with the experimental results.

4. Cracking stress of randomly reinforced composite

The fibre glass-reinforced inorganic cement

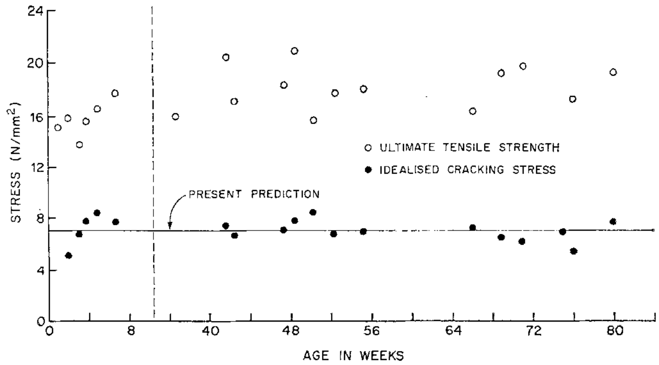

Figure 7 Cracking stress of randomly-reinforced glass fibre cement sheet.

envisaged for constructional use is a composite, reinforced randomly with short fibres, e.g., the thin sheet material examined by the authors previously [1]. The present hypothesis for predicting cracking stress can also be applied to this case. It is assumed that, since the thickness of the material is relatively small compared with the length of the fibres, all fibres are parallel to the two faces of the sheet and are randomly distributed in a two-dimensional space. It has been shown by Krenchel [8] that the efficiency factor for random distribution is

$$n_\phi = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} \cos^4 \phi d\phi = \tfrac{3}{8} \quad (6)$$

The efficiency factor for short fibres is

$$n_l = 1 - 2\frac{l_s}{l} \quad (7)$$

where l_s is the length required for full anchorage and l is the length of the fibres used.

The total efficiency factor for randomly distributed short fibres is

$$n = n_\phi n_l \quad (8)$$

n represents the proportion of fibres which is effective in resisting stress in any particular direction.

If the volume fraction of fibre reinforcement is V_f and the cross-sectional area of each fibre is A, then the effective number of fibres N, per unit cross-sectional area of composite will be

$$N = \frac{n V_f}{A} \quad (9)$$

Since the fibres are randomly distributed it can be assumed that the number of effective fibres will be evenly spaced. Thus the effective spacing will be

$$s = \frac{1}{\sqrt{N}} \quad (10)$$

substituting value of s from Equation 10 into Equation 4 the cracking stress can be obtained.

As an example, the sheet material used in Ref. [1] is examined. The fibres used are 2 in. (50.8 mm) long and each is composed of 204 filaments of 3.8×10^4 in. (9.65×10^{-3} mm) diameter. The volume fraction of fibres is 12.6%. It was found that l_s would not be greater than 3 mm [9]. From Equations 6, 7, and 8, $n = 0.33$.

The effective number of fibres estimated from Equation 9 is 1.83×10^3 per in.^{2} (2.79 per mm^2). The effective spacing s of the fibre from Equation 10 is then 2.34×10^{-2} in. (0.6 mm). Using this value of s in Equation 4 gives a cracking stress, σ_0, of 926 psi (6.4 N/mm^2), which is in good agreement with the results in Ref. [1] (see Fig. 7).

Conclusion

A method for predicting the cracking stress of glass fibre-reinforced inorganic cement based on the fracture-arrest concept was proposed and found to be in good agreement with experimental results.

Acknowledgement

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